Semester 1 Maths Megadoc

Contents:

[> emdcollection.github.io <](emdcollection.github.io)

# Surds

Definitions:

|  |  |
| --- | --- |
| Rational Numbers | Numbers that either terminate (1, 5.6 etc) or are recurring (). They can be represented by fraction or decimal. |
| Irrational Numbers | Numbers that don’t terminate or reoccur. This include surds, as well as π. They cannot be represented by fraction or decimal. |
| Root (or Radical) Sign (√) | The sign that accompanies radicands (). |
| Radicand | Number underneath the square root. |

Basics on Surds

A surd is a representation of what number needed to be squared to reach the radicand.

Simplification of Surds

To simplify a surd, split the surd into two of its factors, with one factor being square-able, resulting in a rational surd multiplied with an irrational surd.

Example

Squaring Surds

A surd to the power of the number above the root sign is the radicand.

A radical sign can be rewritten as the radicand to the power of ½.

Dividing Surds

Rationalising the Denominator

When rationalising the denominator, you need to multiply the fraction by the denominator over itself. Remember that the denominator over itself is equal to 1.

When the denominator contains two terms (an integer and a surd) you can rationalise the denominator by multiplying the numerator and denominator with the conjugate of the denominator. This is not the case when the integer is being multiplied with the surd ().

Multiplying, Adding, and Subtracting Surds

Multiplying, adding, and subtracting surds is the same as that with algebraic terms.

Where to find the content?

Courses on Surds can be found on Jacplus (1.3-1.4) and Oxford (Year 10: 3A-E)

# Indices

Definitions:

|  |  |
| --- | --- |
| Index (Indexes and Indices are interchangeable, so is Power and Exponent) | The exponent to which the base is raised. |
| Base | The number being raised by the exponent. |

Basics of Indices

When a term has an index, it is equal to that term multiplied by itself x amount of times, where x is the value of the index. The term that has the index is called the Base, and the index is also known as the Power. In this case, a is the base, and 3 is the index or power.

Index terms act like algebraic terms, and add and subtract in the same way.

1st Index Law: Multiplying Indices

When like terms are multiplied, and both terms have indexes, the result is the base with the index of the original terms added.

2nd Index Law: Dividing Indices

The opposite applies to like terms divided with each other. In this case, 3 – 2 = 1, so the top base is a to the power of 1. When a base gets “minus-ed out” like the denominator, it becomes 1 (See 3rd Index Law: Power of 0).

A better way to represent this is without fractions.

3rd Index Law: Power of 0

When a number has an index of 0, it is equal to 1.

This is because the above equation can be rewritten like the below. Since a to the power of 0 is the product of no numbers, the result is 1.

Indices and Brackets

When both the term inside the brackets and the brackets itself have indexes, they are multiplied.

Negative Indices

When the index is negative, the result is 1 over the number that comes from the same number with a positive index.

If the index is negative on the denominator, it can be flipped to become the numerator, which makes the index positive. Naturally, the opposite is true for the numerator.

Relationship with Surds

When an Index is in a surd, it is divided by two. This is due to a square root being rewritten as such:

The number to the left of the radical sign (also called an index) replaces the above examples “2.” For example:

This relates back to Indices in Brackets.

An index can be rewritten into a surd if it is a fraction. The denominator is the index of the surd, and the numerator is the index of the radicand, which would be the base.

Where to find this content?

Courses on Indices can be found on Jacplus (1.5-1.9) and Oxford (Year 10: 2A-B, 3F)

# Logarithms

Basics on Logs

An index term can be transitioned into a logarithm using the below formula:

Logarithms find what the number (a) was raised to (b) to get to the product (c).

1st Log Law

Two logs with the same base can multiply bases when adding.

2nd Log Law

Two logs with the same base can divide bases when subtracting.

3rd Log Law

If a log has an exterior index (an index applying to the log), the log can be multiplied by that index.

4th Log Law

Any log with a product of 1 is equal to 0. This is because anything to the power of 0 is equal to 1.

5th Log Law

If the base and the product are the same, the result is 1.

6th Log Law

If the product of a log is a number over 1, the log changes to separate the two logs.

Negatives don’t affect logs.

7th Log Law

If the product has an index, the result is the value of that index. This assumes the base of the log and the base of the index are the same.

Where to find this content?

Courses on Logarithms can be found on Jacplus (1.10-1.12) and Oxford (Year 10: 3G-H)

# Pythagoras and Trigonometry I

Definitions

|  |  |
| --- | --- |
| Adjacent | The side of a triangle that is adjacent to the angle, but isn’t the hypotenuse (not the largest side) |
| Opposite | The side of the triangle not adjacent to the angle (only adjacent to the 90\* angle). |
| Hypotenuse | The largest side of the triangle. |
| Theta | unknown angle, or just the angle. |
| Pythagoras’ Theorem | Theorem of the sides of triangles. When you know two sides of the triangle, you can use the formula. |

What is Pythagoras’ Theorem?

Pythagoras’ Theorem relates to the sides of a triangle. It states:

In this case, a and b are the adjacent and opposite (they are interchangeable in this case), and c is the hypotenuse (not interchangeable, c always must be the hypotenuse). Using Pythagoras’ Theorem, we can figure out that:

Trigonometric Functions

When you need to find a side of the triangle, but only have the value of one side, as well as an angle, you can use trigonometric functions. Remember:

Take the below triangle:

A triangle with a point and points

Description automatically generated with medium confidence

We can find out the adjacent side and the hypotenuse. Let’s start with the adjacent.

We know the opposite side, and we’re looking for the adjacent side. We can use tan, as that function deals with the opposite and adjacent side.

Let’s plug our numbers into this equation.

We can rearrange this to find a. First, we multiply both sides by a.

Then, we can divide both sides by tan (57).

Now, we can use our calculator to find a.

Now, we can look for the hypotenuse as well. This is done in the same way, but instead of using tan, we use sine.

Since the unknown value is again on the bottom, we can again use the same formula:

There are our sides! Notice how the hypotenuse is the largest side.

Finding the Angles

If we know two of the sides, we can figure out the angles. We need to use arcsine, or sine to the power of -1. Let’s use the opposite and the hypotenuse (our rounded-up versions).

Since we already knew one angle, and the other angle is 90\*, we have found every angle and every side of the triangle. Now, we can perform relevant calculations.

# Circle Geometry

A table with text and circles

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A diagram of a circle with text

Description automatically generated with medium confidence

Circle Theorems

|  |  |  |
| --- | --- | --- |
| 1 | The angle subtended at the centre of a circle is twice the angle subtended at the circumference standing on the same arc.  In the circles to the right, cords AC and CB form angle ACB, which means arc AB has subtended the angle ACB.  Angle PRQ is half of angle POQ | c14f16c14f18 |
| 2 | All angles that have their vertex on the circumference and are subtended by the same arc are equal.  All of the angles in the circle to the right (that start at P, go to a point on the circumference | c14f21 |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 | The opposite angles of a cyclic quadrilateral are supplementary (add to 180 degrees).  ABC + ADC = 180  DAB + DCB = 180 | c14f86 |
| 12 | The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.  QRS + SPQ = 180  TPS + SPQ = 180  Therefore, TPS = QRS | c14f91 |
| 13 | The angle between a tangent and a chord is equal to the angle in the alternate segment.  BAD = AFD | c14f110 |
| 14 | If a tangent and a secant intersect as shown, the following relationship is always true:  AB = C2 | c14f115 |

# Trigonometry II

Sine Rule:

* The sine values of supplementary angles are equal – eg:
* The sides of a **non-right-angle triangle** are labelled in **lowercase letters,** whilst the **angles** are labelled in **uppercase letters.**
* These letters are done so with **a, b, and c for sides**, and **A, B, and C for angles.**

The Sine rule states that:

* The sine rule can be used to find unknown parts of a triangle, where there are at least 3 values already known, and two of which are corresponding angle to side lengths, eg: A and a, or B and b, or C and c.

Area formula:

* All letters must be different.
* The angle must be located inside the two sides listed.
* In this formula it is shown that the and “A” is located between sides “b” and “c”.

Sine Rule Examples:

A triangle with numbers and a triangle

Description automatically generated

b

c

A

C

a

B

Step 1: Label sides and Angles:

Step 2: Identify common side and angle:

Common side and angle are C and c as both values for that side are known.

Step 3: Plug into sine rule:

Step 4: Solve for pronumeral:

* If pronumeral is on bottom (in this case it is), bring all of left side ()to denominator of right side and bring denominator of right side, in this case () over to left side (example shown below).
* If pronumeral on the top, then multiply denominator on both sides to isolate pronumeral.

Step 5: Isolate B (if necessary):

Step 6: Plug into calculator:

# Bearings

What are bearings?

A circle with arrows and a circle in the middle

Description automatically generated

Bearings, or compass bearing, are a way of measuring the angle between a ling and a compass grid axis, like the grid above. There are two types of bearings:

* Compass Bearings
* True Bearing

Both share generally the same concept.

Compass Bearings

Compass bearings are written as such. First, N or S (which ever is closer, so if the line is above (N) or below (S) the x-axis. Then is the number in the middle, which is an angle. The angle is the angle the line makes with either the North or South line. So basically, the angle the line makes with the y-axis is what angle should be in the middle. Finally is E or W. This, again, depends on whether the lines falls on the left (W) or right (E) side of the y-axis.

Take the below graph.

A blue circle with black arrows

Description automatically generated

The angle the line makes with the y-axis is 45 (just for this example). It falls on the northern side because it is above the x-axis, and on the eastern side because it is to the right of the y-axis. This means this bearing is written out as **N 45\* E**.

True Bearings

The only difference between True bearings and Compass Bearings is that the angle is always measured from the northern part of the y-axis, clockwise, and there is no east or west written out. Also, the angle is always written out in three digits. For example, if the true bearing angle was 45, it would be written as 045.

Let’s take another graph.

A circle with arrows and a point

Description automatically generated

The line in this graph has a compass bearing of **S 70\* W**, but the question asks for the true bearing, not the compass bearing. To convert the bearings, measure the angle from the northern part of the y-axis to the line. You can just add 90\* for every quadrant the line isn’t in clockwise. For this example, the true bearing angle would pass through the two rightmost quadrants, but since it isn’t in there, we can add 180 (90 \* 2). Since the angle 70\* was measured from the y-axis, we can simply add it to our 180\*. This means the true bearing of the line above is **250\***.

To write out a true bearing, write the angle followed by T. The above graph has a bearing of **250\* T**.

Where to find this content?

Content on Beari ngs can be found on Jacplus (Year 10: 5.8) and Oxford (Year 10: 8C).

# Linear Equations and Quadratics

Definitions

|  |  |
| --- | --- |
| Quadratic | A linear equation where the pronumeral with the highest power is . |
| Monic | One (the leading coefficient is 1 () |
| Trinomial | 3 Terms (the squared pronumeral, the pronumeral with the coefficient, and the constant) |
| Constant | A number with no pronumeral. The third number in a quadratic formula. |

What are linear equations?

Usually, a linear equation is set up like the below:

To solve it, you need to move some of the numbers around. For example, say we need to get the 2x on the left side. Since the 2x on the right side is positive, we would need to subtract 2x. But then the equation wouldn’t be true, because the two sides wouldn’t be equal.

When a change is made in linear equations, it is made to both sides. So, we need to subtract 2x from both sides.

Now, we need to move the 14 to the right side. This is done in the same way: subtracting 14 from both sides.

Now simply multiply both sides by -1, so that x is positive.

Substitution Method

Usually, linear equations look more like this:

In this case, the goal is to find a coordinate (x,y). There are multiple ways to go about this. First is the substitution method. Since both of the above equations are equal to y, they also equal each other.

Now we solve for x like normal. First we add 2 to both sides, and then subtract 3x from both sides.

Then we just divide both sides by 4 and we get our x value, which can remain a fraction.

Now that we have an x value, we can substitute it into one of the initial equations.

Elimination Method

The first elimination method involves adding the two equations together.

Now we can solve for x.

Now we substitute our new x value into one of the two original equations.

This technique only works when the two terms with identical pronumeral and coefficient are opposite in sign of each other. If the coefficients were both positive or both negative, we would subtract the two equations together.

If the coefficients are different, then you can multiply the whole equation, so the coefficients match.

In the above equation, all we would need to do is multiply the first equation by 2 (this involves multiplying everything by 2, which includes the coefficient belonging to y and the number to the right of the equal sign).

We wouldn’t want to multiply the second equation by 4/3 or the first equation by 3/4, because the answer and the other coefficients wouldn’t look to pretty. In this instance, we multiply the first equation by 3 and the second equation by 4. This way, the first coefficients match and there are no decimals.

What are quadratics?

Quadratics are a special kind of linear equation. There are two types:

Quadratic formulas are made up of a pronumeral with a power of 2, a pronumeral with no powers and a coefficient (which can be 1 or a negative number), and a constant, being the last number with no pronumeral.

How do I solve monic quadratics?

Quadratics have two answers, which relates to how we factorise them. We first should list all of the factors of the constant.

Now, we find the two matching factors which add up to the coefficient of the second pronumeral, which in this case is 9. The only pair that equals 9 is 10 + (-1).

And there is our answer! We take the two factors and set them up as such:

However, most quadratic problems will require more work. Most quadratic problems have something extra:

This is where the Null Factor law comes in. The null factor law states:

Therefore:

To find our two x values, we can multiply each of the constants by -1. This is because:

And:

Solving Non-monic quadratics

This can be done in multiple ways (See [this website](https://geomaths.co.uk/2020/01/09/factorising-non-monic-quadratics-a-departments-view/) to learn more methods not talked about here). This doc only talks about one method in detail.

First, draw a big X. Put the factors of (the squared pronumeral) on the top left and bottom left (the only factors possible are 3x and 1x), and then the factors of 3 (the constant) on the top right and bottom right (the only factors possible are 1 and 3). The goal is that the two numbers connected by lines should be multiplied, and that both pairs of lines should add up to the term in the middle (10x).

3x

x

3

1

With this setup, we multiply 3x by 3, and 1x by 1, which equals 10x when added together. This can be written as such:

Now, we set up the brackets so that both of the top numbers are in a bracket, and both of the bottom numbers are in a bracket.

Expanding (Proofing) a Factorized Quadratic

How do we know all of these answer are correct? Simply perform the equation we are left with (multiply the two brackets with each other). This is done using the FOIL method.

First

First you multiply the outside term (the first term in the first bracket) with the two terms in the second bracket individually. Then, you do the same with the inside term last (the second term in the first bracket).

Using the above example, we get the below:

Look at that! It’s the same as the starting equation. This means our factorised form is true. But what about the non-monic equation?

Using the FOIL method again, we get:

It’s the same again! Our non-monic factorization is true as well.

Where to find this content?

Courses on Linear Equations can be found on Jacplus (Year 10: All of Chapter 4) and Oxford (Year 10: All of Chapter 5). Courses on Quadratics can be found on Jacplus (Year 10: All of Chapter 7 and 8) and Oxford (Year 10: 5B).

# Inequalities and Linear Graphs

|  |  |
| --- | --- |
| X-intercept | Where the line or graph intercepts through the x-axis. Represented as a coordinate, which is written as (x-intercept,0) |
| Y-intercept | Where the line or graph intercepts through the y-axis. Represented as a coordinate, which is written as (0,y-intercept). Represented as the letter c. |
| Gradient | The slope of the line (how steep or gentile it is). Calculated using , where the rise is how much a point goes up or down on the y-axis, and the run is how much a point goes left or right on the x-axis. The graph 2x has a rise of 2 and a run of 1. This means that when the x value increments by 1, the y value increments by 2. |
| Coordinate | A point on the cartesian plane, represented with (x,y). |
| Turning Point | The point at which the y value reaches its minimum or maximum; the bottom of the curve. |

What are Linear Graphs?

We can graph linear equations, assuming a linear equation is equal to y, or if both an x and y pronumeral are present.

A linear non-quadratic line is written commonly in one of three styles.

Standard Line

This is the standard formula for a line. To figure out how to draw lines using this formula (say the question restricts calculators) you need to bring the y pronumeral to a side on its own, and divide it until it has a coefficient of 1 (remember, changes occur on both sides).

Take the following standard line:

First, we bring the y to the right by subtracting 6y from both sides.

Next, we add 4 on both sides.

Next, we multiply each side by -1, to make the y coefficient positive.

Finally, we divide each side by 6.

If we plot this on a graph, we get this:

A graph of x and y axis

Description automatically generated

All of those values influence the lines, as seen with the slope-intercept line.

Slope-Intercept Line

This is the equation for a slope-intercept line, which is the line we want to convert the standard line into. In this formula, y represents the pronumeral y (or f(x)), m represents the gradient, and c represents the y-intercept.

Take this line:

Without even drawing a graph, we can tell that it has a gradient of 1 (when its x value increments by 1, the y value increments by 4), and its y-intercept is -9. We can even get the x-intercept by dividing -9 by 4 and multiplying it by -1, which gives the x-intercept.

With this graph, we know:

This is all we would need to know about any line, unless we need specific information about what y would be at a specific x value.

Point Line (Finding a Line using the Gradient and a point)

If you are only given a gradient and a point that the line passes, you can find the equation for that line using the below formula:

In this case, y1 and x1 are the y and x coordinates of the point.

Say we have a gradient of 0.5 and the point (4,5). We know what y1 is, so we can move it to the right. Our equation is now:

We get a line that looks like this:

A graph of a line and a point

Description automatically generated

The line goes through (4,5) (marked in red), and it has a gradient of 0.5.

Find other values on a line

To find the midpoint of two points, use the below formula:

To find the gradient of two points, use the below formula:

To find the distance between two points, use the below formula:

Inequalities

Inequalities are linear equations that aren’t equal to each other (the = is replaced with a <, >, <=, or >=). They are solved like regular linear equations.

However, when they are represented with a graph, they appear different.

A graph of a function

Description automatically generated

y < x

Since y has an infinite number of solutions for each x value, given that its value is only defined as being less than x, we represent all of its possibilities by shading the space. The line is dashed because of the less than sign. The line shows that y cannot be any point on the line.

A graph of a function

Description automatically generated

y ≤ x

Now the line is solid, because y can be any point on the line, unlike before.

A graph of a function

Description automatically generatedA graph of a line drawn on a grid

Description automatically generated

y ≥ 2x + 4 and y = 2x + 4 respectively

Other than shading a specific region above/below the line, nothing else changes with how you draw the line.

Non-linear graphs

A graph of a function

Description automatically generated with medium confidence

Parabola in blue, Hyperbola in red, Circle/Elipse in orange, Quartic in purple.

Quadratic equations can also be graphs, as well as other graphs. Quadratic graphs are known as parabolas.

A graph of function on a grid

Description automatically generated

y = 2x^2 in blue, y = x^2 – 6 in red, y = 0.5x^2 in purple

Let’s take a previous quadratic equation, just to see what values influence the parabola and how.

A graph of x and y axis

Description automatically generated

The 3 in front of the x squared changes the width of the parabola (in this case, 3 makes it thinner, just as a greater gradient would make a line thinner). If the three was negative, the parabola would be flipped. The constant 3 is the y-intercept. The 10 influences where the turning point of the parabola is. To find it, we use this formula:

In this case, a is the 10 in 10x, and b is the “gradient” 3. This means the turning point is at 0.5 \* 10 / 3, or 5 / 3, which is ~1.6667.

The “answer” to this graph would be the two x-intercepts. Fortunately, we already know the answers to this quadratic expression.

Looking at the parabola, the x values are the same.

A graph paper with a line in the center

Description automatically generated

Hyperbolas

A graph of a function

Description automatically generated

Hyperbolas are the result of y being a number over x.

A and b both change how close the line is to the centre of the graph (the midpoint). The “turning point” of the foci can be represented by the below coordinate:

The term dx changes the “gradient” of the hyperbola. Usually, value d is equal to 0.

A graph of a function

Description automatically generated

Pronumerals c and e change the x and y position of the midpoint of the hyperbole. The coordinate of the centre can be represented by the below formula. Note that c is negative, or multiplied by -1.

Usually, the “answers” of a hyperbola will be its x-intercept with another line.

Circles

A graphing of a function

Description automatically generated with medium confidence

Circles are graphs represented by the below formula:

Pronumerals a and b act as “gradients.” They change the width of the circle on the x and y axis. C and d both effect the diameter of the circle. To see this, remember that (if the centre of the circle is at (0,0) and a and b are both 1) that the formula to find the top of the circle is this:

To find the bottom, just make the y value negative, and to find the left and rightmost side, swap the 0 and square root.

Quartics

A graph of x and y axis

Description automatically generated

x^3 in red, x^4 in blue

Quartics are graphs that occur when the x is raised to the power of >2. The formula is as such:

Pronumeral a controls the width of the quartic. b controls how steep the quartic is. c controls the y-intercept.

Piecewise

Piecewise are lines comprised of multiple equations. They are written like such:

A graph of a function

Description automatically generated

Just like the solid and dashed lines, the solid dots represent points which are part of the piecewise, and the hollow ones represent points that aren’t part of it.

Where to find this content?

Courses on everything here can be found on Jacplus (Year 10: All of Chapter 4 and Chapter 9) and Oxford (Year 10: All of Chapter 5 and 4B-F).

# [Useful Ti-Nspire Functions](https://paradevic.sharepoint.com/sites/msteams_f28f5e/Shared%20Documents/General/TUTORIALS%20FOR%20THE%20TI.pdf)